Fluctuations and Dissipation of Coherent Magnetization

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Abstract

A quantum mechanical model is used to derive a generalized Landau-Lifshitz equation for a magnetic moment, including fluctuations and dissipation. The model reproduces the Gilbert-Brown form of the equation in the classical limit. The magnetic moment is linearly coupled to a reservoir of bosonic degrees of freedom. Use of generalized coherent states makes the semiclassical limit more transparent within a path-integral formulation. A general fluctuation-dissipation theorem is derived. The magnitude of the magnetic moment also fluctuates beyond the Gaussian approximation. We discuss how the approximate stochastic description of the thermal field follows from our result. As an example, we go beyond the linear-response method and show how the thermal fluctuations become anisotropy-dependent even in the uniaxial case.

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1 INTRODUCTION

The study of thermally induced magnetization reversal was first carried out by Brown (1). His approach was to introduce a noise term into the Landau-Lifshitz equation of motion, essentially constructing a Langevin-type equation, which we call here the Landau-Lifshitz-Gilbert-Brown equation (LLGB). From the LLGB equation the Fokker-Plank equation can be derived that describes the time evolution of the probability density distribution of the moment orientations. Solution of this problem was carried out by Brown for the case of an axially symmetric potential and later by Coffey et al (2) for non-axially symmetric cases. The numerical solution of the Langevin equation was used by Lyberatos et al (3), and has since been applied to the study of magnetization reversal by a number of authors, (4; 5). Recently, Wang et al (6) have developed an approach introducing a tensor form of the damping constant and applied this to the calculation of first mean passage time in the case of an elongated grain represented as a chain of coupled particles. Many of these calculations are motivated by the need to understand high frequency magnetization reversal processes in magnetic recording. The process of reading information currently involves GMR sensors, the size of which is continually reducing as recording densities increase. This led Smith and Arnett (7) to the conjecture that noise due to magnetization fluctuations in the read head would be a limiting factor on the device size. This is clearly an important problem, which has been further developed by Smith (8) and Bertram and co-workers (9), who have also studied the full micromagnetic description of the problem using an approach in which the thermal noise is distributed among the normal modes (10).

Clearly the introduction of thermal fluctuations in the micromagnetic formalism is important both from the point of view of the physics of magnetization processes and also in relation to important practical problems of magnetic recording. Central to all models, both analytical and numerical, is the introduction of a magnetization fluctuation or an effective field via the fluctuation-dissipation theorem (FDT) (11). The FDT has a strong physical justification, as discussed in detail by Landau and Lifshitz (12), however it should be stressed that it is strictly valid only for small fluctuations about the local minimum. A more serious problem with the use of the Langevin equation in the LLGB form is the dissipative term itself, which has no microscopic justification. It is clearly important to understand the whole problem of fluctuations and dissipation within a first priciples quantum mechanical approach, if the limits of the current models are to be established and more fundamental theoretical approaches derived. The demand for higher density recording media and faster switching rates requires the use of structures on the nanometer scale or less. Quantum mechanical effects are then bound to become more and more important to consider in these systems. Effects such as magneto-optical interactions may even invalidate the simple damping term that is currently used in the Landau-Lifshitz equation. This prompted us to investigate whether the LLGB

equation can be recovered from a more fundamental treatment rather than the ad hoc approach presently used. Hence it seems natural to ask in what limit the LLGB equation can be recovered starting from a quantum model.

In this paper, we make a small step toward that goal. To address the above questions, we take a simple quantum model, that of a single particle with large spin interacting with a heat bath and an external magnetic field. The spin is taken to be large since we are primarily interested in a semiclassical representation of the magnetization vector. This simple model is sufficient to allow us to study the effects of thermal fluctuations in many different cases, such as magnetization switching in a single domain magnetic particle with uniaxial anisotropy, an important problem in magnetic recording physics. The bath is taken to be of bosonic nature. Nothing else needs to be assumed to enable us to include various mechanisms of interaction between the magnetic moment and the environment. We calculate the equations of motion of the magnetization and that of an associated fluctuating field in the semi-classical limit. Since our interest is mainly in the semi-classical limit, coherent states (CS) are the natural choice for the representation of the system. These states have the property of minimizing the Heisenberg uncertainty relations. Furthermore, the calculations are based on expressing the density matrix of the particle-bath system in terms of path-integrals as in the Feynman-Vernon formalism (13; 14; 15). Clearly this method allows a consistent treatment of the magnetization and fluctuations from the start. If the thermal field is decoupled from the magnetization, the LLGB equation will be shown to correspond to a given choice of density of states of the reservoir and of its interaction parameters with the magnetic moment.

This work is able to provide a different angle from which to discuss the LLGB equation and its limitation. We also set a basis against which we can examine the discrepancy in the recent calculations of the noise spectrum in magnetic recording heads (7; 10). Therefore, a treatment of the noise problem by a self-consistent method, such as the one presented here, can shed some light on why this difference exists. We stress that our results are very general for the model considered and no linear approximation is assumed.

The paper is organized as follows. In section two, we introduce a simple model Hamiltonian that can describe dissipation and fluctuations. We linearly couple a single domain magnetic particle to an external magnetic field and to a Bosonic bath with infinite number of degrees of freedom. In section three, we show how to calculate the reduced density matrix elements of the magnetic moment. The density matrix elements are naturally expressed in terms of path-integrals over the configuration space of the moment (14). In section four, we derive coupled equations for the magnetization and fluctuations. We show that a general fluctuation-dissipation theorem is satisfied. We also demonstrate how to recover the LLGB equation by decoupling the thermal fluctuations, taking the high temperature limit and constraining the choice of reservoir. In the particular case of LLGB, this corresponds to Gaussian fluctuations and constant

dissipation. Similar results have been obtained for the case when the magnetic moment is replaced by a harmonic oscillator. This is no surprise since in this case, the semi-classical approximation corresponds to a particle with large spin. In section five, we compare the classical stochastic treatment to this quantum treatment. As an example we calculate the fluctuating field for a single domain particle with the external field directed along the easy axis as a function of the anisotropy constant. Finally in the last section, we summarize our results and state results based on our derived equations when generalized to the anisotropic case.

2 DEFINITION OF THE MODEL

The model we choose is simple but general enough to include many interesting physical situations. It is mainly motivated by the recent work of Safonov and Bertram (16). They used a two-level impurity system to simulate relaxation effects in a single domain grain. They showed that the damping in their model is of the Gilbert form. No fluctuations are considered in their calculation. If we consider a collection of spins that are independent, then the magnetization vector, $\mathbf{M}$, is a simple sum of these coherent spins,

$$\mathbf{M} = g\mu_B \frac{N\mathbf{S}}{V}, \tag{1}$$

where $\mathbf{S}$ is the spin vector and $g\mu_B/\hbar$ is the gyromagnetic ratio. μ_B is the Bohr magneton and V is the volume of the system. In the following we set $\hbar = 1$, $g\mu_B = 1$ and the density $\frac{N}{V} = 1$. From now on, we use the words spin and magnetic moment interchangeably.

We take a single spin $\mathbf{S}$ $\left(\mathbf{S}^2 \gg 1\right)$ and couple it linearly to a set of oscillators and to a constant external field $\mathbf{H}$. The former may represent phonons, a time-dependent magnetic field or other Bosonic degrees of freedom. No assumption will be made about the coupling constants or the density of states of the reservoir. The Hamiltonian assumes the following form:

$$\mathcal{H} = -\mathbf{H}\cdot\mathbf{S} + \sum_k \omega_k a_k^+ a_k + \sum_k \gamma_k a_k^+ S_- + \sum_k \gamma_k^* S_+ a_k \tag{2}$$

where $\mathbf{H}$ is a static external magnetic field. $\mathbf{S}$ is the magnetic moment of a single particle. a_k^+ and a_k are creation and annihilation operators of the reservoir. The coupling constants γ_k may be time-dependent, but will be taken as independent of time in the final result. The field $\mathbf{H}$ is taken along the z-axis, the axis of spin quantization. Coupling the z-component of the vector $\mathbf{S}$ to the reservoir can be easily added, but it will be omitted in this work. This Hamiltonian is sufficient to describe all the desired physics. Using the equation

of motion for S_z, it is trivial to see that it is not a constant of the motion and hence no linearization is implied in this model.

The operators are in the Heisenberg representation. The spin operator S satisfies the usual commutation relations (17)

$$\left[\mathbf{S}^2, S_\pm\right] = 0 \tag{3}$$

with

$$\mathbf{S}^2 = \frac{1}{2}\left\{S_+, S_-\right\} + S_z^2, \tag{4}$$

where the curly brackets are for anticommutation and

$$S_+ = S_x + iS_y \tag{5}$$

$$S_- = S_x - iS_y \tag{6}$$

while the operators of the reservoir satisfy Bose commutation relations,

$$\left[\ a_k, a_{k'}^+\right] = \delta_{kk'}\ . \tag{7}$$

Instead of the usual Fock space representation, we use a CS space representation for these operators (18; 19; 20) .

For a harmonic oscillator with position q_k, momentum p_k and frequency ω_k, the CS $|\Phi_k\rangle$ are defined as eigenfunctions of the annihilation operator $a_k = \left(\frac{\omega_k}{2}\right)^{1/2}\left(q_k + \frac{i}{(2\omega_k)^{1/2}} p_k\right)$

$$a_k|\mathbf{\Phi}_k\rangle = \Phi_k|\mathbf{\Phi}_k\rangle \tag{8}$$

with complex eigenvalues, Φ_k (18). These states can also be generated from the ground state by applying a displacement operator $D\left(z_k\right)$ which defines a one-to-one correspondence between the complex plane and the oscillator states,

$$|z_k\rangle = D\left(z_k\right)|0_k\rangle \tag{9}$$

and

$$D\left(z_k\right) = \exp\left(z_k a_k^+ - z_k^* a_k\right). \tag{10}$$

CS's form an overcomplete basis and satisfy the minimum uncertainty relation. Hence they are the most suitable representation for a semi-classical treatment. We also adopt the normalization in (19)

$$\langle\Phi_k|\Phi_k'\rangle = e^{\Phi_k^*\Phi_k'}. \tag{11}$$

They also satisfy the following relation, the resolution of the identity operator,

$$\int \frac{d\Phi_k^* d\Phi_k}{2\pi i} e^{-\Phi_k^*\Phi_k}|\Phi_k\rangle\langle\Phi_k| = 1. \tag{12}$$

The latter relation is essential for a path-integral representation in terms of CS's.

Similarly for the spin states, we use a CS representation (21; 22). They are defined by analogy to the harmonic oscillator CS's. The spin components in this state satisfy a minimum uncertainty relation, i.e., two of the three components commute (23). As in the harmonic oscillator case, a 'ground' state $|0\rangle$ is required from which to generate all the other states. In this case the state with the largest S_z component is taken as the reference state. If the z-axis is taken as the quantization axis and if we take $\mathbf{S}^2 = j\,(j+1)$, then by definition, we have

$$|0\rangle \equiv |j, j\rangle, \tag{13}$$

and

$$S_z|0\rangle = j|0\rangle \tag{14}$$

i.e., the state with the minimum fluctuations (17). The spin CS's are a generalization of the Holstein-Primakoff construction (24). They are defined in terms of deviations from the maximum positive z-component of the spin $\mathbf{S}$

$$S_z|\mathbf{p}\rangle = (j-p)\,|\mathbf{p}\rangle\ . \tag{15}$$

The CS's are then constructed using

$$\begin{aligned} |\mu\rangle &= \frac{1}{\left(1+|\mu|^2\right)^j} \exp\left(\mu S_-\right)|0\rangle \qquad (16) \\ &= \frac{1}{\left(1+|\mu|^2\right)^j} \sum_{p=0}^{2j} \left(\frac{(2j)!}{p!\,(2j-p)!}\right)^{\frac{1}{2}} \mu^p |\mathbf{p}\rangle \end{aligned}$$

where μ is a complex number. Since the configuration space of $\mathbf{S}$ is the surface of a sphere, it will be clearer to have μ parametrize the surface of a sphere through a stereographic projection,

$$\mu = \tan\left(\frac{1}{2}\theta\right) e^{i\varphi}. \tag{17}$$

In this representation, a CS will be represented by a solid angle $\mathbf{\Omega}$:

$$|\mathbf{\Omega}\rangle = |\theta,\varphi\rangle = \left(\cos\frac{1}{2}\theta\right)^{2j} \exp\left\{\tan\left(\frac{1}{2}\theta\right) e^{i\varphi} S_-\right\}|0\rangle\ . \tag{18}$$

A useful property for a path integral formulation is that the unit operator has the familiar decomposition in terms of projection operators on all CS's,

$$\frac{2j+1}{4\pi}\int d\mathbf{\Omega} \quad |\mathbf{\Omega}\rangle\langle\mathbf{\Omega}| = 1. \tag{19}$$

In this representation, the overlap of two coherent states represents an area on a sphere, the surface of which is the configuration space of the spin $\mathbf{S}$. The overlap is

$$\langle\, \mathbf{\Omega}' | \mathbf{\Omega}\, \rangle = \left\{ \cos\frac{1}{2}\theta \cos\frac{1}{2}\theta' + \sin\frac{1}{2}\theta \sin\frac{1}{2}\theta' e^{i\left(\varphi - \varphi'\right)} \right\}^{2j} \tag{20}$$

and its magnitude is

$$|\langle\, \mathbf{\Omega}' | \mathbf{\Omega}\, \rangle| = \left(\frac{1 + \mathbf{n}\cdot\mathbf{n}'}{2} \right)^{j} \; . \tag{21}$$

Since we plan to use a path-integral technique, we need to write the expectation values of the Hamiltonian in the coherent representation. These expectation values follow in turn from those of the operators S_z, S_+ and S_-. The following expectation values are deduced from Eq.(18) and Eq. (20),

$$\langle\, \mathbf{\Omega} | j - S_z | \mathbf{\Omega}\, \rangle = j\,(1 - \cos\theta)\,, \tag{22}$$

$$\langle\, \mathbf{\Omega} | S_+ | \mathbf{\Omega}\, \rangle = j \sin\theta e^{i\theta}, \tag{23}$$

$$\langle\, \mathbf{\Omega} | S_- | \mathbf{\Omega}\, \rangle = j \sin\theta e^{-i\theta}, \tag{24}$$

$$\langle\, \mathbf{\Omega} | \mathbf{S} | \mathbf{\Omega}\, \rangle = j\mathbf{n}. \tag{25}$$

$\mathbf{n}$ is a unit vector with angles (θ, φ). For $j \gg 1$, the off-diagonal terms of the spin operator are smaller than the diagonal ones by a factor of about $\sqrt{j}$. Hence they are negligible in the classical limit. This limit will be implicit in all subsequent calculations of the reduced density matrix elements.

3 REDUCED DENSITY MATRIX ELEMENTS OF THE SPIN PARTICLE

In the following, we make use of CS for both the bath degrees of freedom and the magnetic moment. The procedure we follow is by now mostly standard. Reference (25) (and references therein) provides a general overview of these methods and hence we omit most of the intermediate steps in our calculation.

Bosonic CS's were first used by Langer (26) to study dissipation and fluctuations in a superfluid, many-body problem. Starting from the equation of

motion of the density matrix of the whole system, ρ, a Landau-Ginzburg equation was recovered in the equilibrium case and a Fokker-Planck equation in the classical limit,

$$i\frac{\partial\rho}{\partial t} = [\mathcal{H},\rho] \ . \tag{26}$$

In a CS formulation, only diagonal elements of the reduced density matrix are needed. Instead of starting from the above equation, we can instead start from an integral representation of the density matrix elements. This method is well known and is based on the Feynman-Vernon formalism (13). This path-integral approach is in real-time as opposed to the imaginary-time approach in equilibrium thermodynamics. Hence questions like approach to equilibrium can be studied within this approach. This method has seen many different applications since the Caldeira-Leggett (CL) work (15). The CL model was successful in showing how to recover the Langevin equation by coupling an oscillator to a bath of oscillators. It seems natural then to ask if the LLGB equation can be recovered by coupling a spin to a bath of oscillators. This important question does not seem to have been addressed in the literature. Spin coherent states are the natural language to answer this question. Hence, we formulate the question in terms of CS and use path-integral techniques to write the density matrix elements of the system. Use of path-integrals with spin CS is not as straightforward as in the case of bosons (27), nevertheless it is the method of choice in this particular problem.

The calculation we present below takes into account the correct boundary conditions as emphasized in ((27)). However we avoid using the more abstract holomorphic representation in favor of a more geometric one, i.e., in terms of solid angles. The physical space for the Hamiltonian , Eq. (2), is the product of the Hilbert space of the spin particle and that of the harmonic oscillators,

$$\prod_k |\mathbf{\Omega}\rangle \otimes |\Phi_k\rangle \, . \tag{27}$$

Using the expectation values of the different operators in the Hamiltonian, we get the expectation value of the Hamiltonian in the coherent representation,

$$\begin{aligned} \mathcal{H}\left[\Phi^*,\Phi,\mathbf{S}\right] &= -H_z j\cos\theta(t) + \sum_k \omega_k \Phi_k^*(t)\Phi_k(t) \\ &\quad + j\sum_k \gamma_k \Phi_k^*(t)\sin\theta(t)e^{-i\varphi(t)} - j\sum_k \gamma_k^*\Phi_k(t)\sin\theta(t)e^{i\varphi(t)}. \end{aligned} \tag{28}$$

From now on, we normalize the magnitude of all spin vectors by j. The reduced density matrix element of the spin particle, $\rho_{ff'}$, is by definition the density matrix element of the whole system averaged over the states, $|\mathbf{\Phi}_k\rangle$, of the bath,

$$\begin{aligned} \rho_{ff'}(t) &= \langle \mathbf{S}_{f'} \ |\rho(t)| \ \mathbf{S}_f\rangle \\ &= \int \prod_k \mathfrak{D}\Phi_k^*\mathfrak{D}\Phi_k \ \langle \mathbf{S}_{f'};\mathbf{\Phi} \ |\rho(t)| \ \mathbf{S}_f;\mathbf{\Phi}\rangle. \end{aligned} \tag{29}$$

where $|\mathbf{S}_f\rangle$ and $|\mathbf{S}_{f'}\rangle$ are two arbitrary CS of the spin.

For simplicity, from now on we use the following notation for the functional measure of the Bosonic degrees of freedom,

$$\int \prod_k \mathfrak{D}\Phi_k^* \mathfrak{D}\Phi_k \;\equiv\; \int \mathfrak{D}\left(\mathbf{\Phi}^*, \mathbf{\Phi}\right). \tag{30}$$

The calculation of density matrix elements is easily carried out using a path-integral representation. The propagator of the Bosonic part can be written in terms of a path-integral (20)

$$\begin{aligned}\langle \mathbf{\Phi}_f \,| e^{-i\int_0^t dt\,\mathcal{H}(t)} |\, \mathbf{\Phi}_i\rangle &= \int_{\mathbf{\Phi}(0)=\Phi_i}^{\mathbf{\Phi}^*(t)=\Phi_f^*} \mathfrak{D}\left(\mathbf{\Phi}^*, \mathbf{\Phi}\right)\, \exp\left\{\sum_k \Phi_k^*\left(t\right)\Phi_k\left(t\right)\right. &(31)\\ &\left. +i\int_0^t dt\left[\sum_k i\Phi_k^*\left(t\right)\partial_t\Phi_k\left(t\right) - \mathcal{H}\left(\mathbf{\Phi}^*, \mathbf{\Phi}, \mathbf{S}\right)\right]\right\} &(32)\end{aligned}$$

Running from an initial time, $t = 0$, to time t, we use a real-time path integral to average over all intermediate states. The density matrix element is then expressed as an integral in terms of the initial density matrix element of the system,

$$\begin{aligned}\rho_{ff'}(t) &= \int \mathfrak{D}\left(\Phi^*, \Phi\right) \int \mathfrak{D}\mathbf{\Omega}_1 \int \mathfrak{D}\mathbf{\Omega}_2 \int \mathfrak{D}\left(\Phi_1^*, \Phi_1\right) \int \mathfrak{D}\left(\Phi_2^*, \Phi_2\right) \qquad (33)\\ &\times\langle \mathbf{S}_f, \Phi; t \mid \mathbf{\Omega}_1, \Phi_1; 0\rangle\langle \mathbf{\Omega}_1, \Phi_1; 0\, |\rho|\, \mathbf{\Omega}_2, \Phi_2; 0\rangle\langle \mathbf{\Omega}_2, \Phi_2; 0 \mid \mathbf{S}_{f\prime}, \Phi, t\rangle.\end{aligned}$$

We make no assumption about the initial state of the spin particle. Hence we have to calculate a forward propagator, a backward propagator and the density matrix element at the initial time. The system is assumed to be at finite temperature. Since the Hamiltonian is quadratic, the integrations are easily carried out in the stationary-phase approximation. We show a few steps in the calculation of the forward propagator. Similar calculations are also done for the other two terms in Eq. (33). The forward propagator is first written as a path integral

$$\begin{aligned}\langle \mathbf{S}_f, \Phi; t \mid \mathbf{\Omega}_1, \Phi_1; 0\rangle &= \left\langle\, \mathbf{S}_f, \Phi \left| e^{-i\int_0^t \mathcal{H}dt} \right| \mathbf{\Omega}_1, \Phi_1 \,\right\rangle \qquad (34)\\ &= \int_{\mathbf{\Omega}_1}^{\mathbf{S}_f} \mathfrak{D}\mathbf{S}_1 \int_{\Phi_1}^{\Phi} \mathfrak{D}\left(\Phi_1^*, \Phi_1\right) \exp\left\{\sum_k \Phi_{1,k}^*(t)\Phi_{1,k}(t)\right.\\ &\quad \left. +i\int_0^t dt'\left[i\sum_k \Phi_{1,k}^*(t')\partial_{t'}\Phi_{1,k}(t') - \mathcal{H}\left(\Phi_1^*, \Phi_1, \mathbf{S}_1\right)\right]\right\}\\ &\quad \times \exp\left\{i\mathbb{S}_{WZ}\left[\mathbf{S}_1\right]\right\}.\end{aligned}$$

The last term is a geometrical term, the Wess-Zumino term (28)(and references therein)

$$\mathbb{S}_{WZ}\left[\mathbf{S}_1\right]=\int_0^1 ds\int_0^t d\tau \mathbf{S}_1\left(s,\tau\right)\cdot\left(\frac{\partial \mathbf{S}_1\left(s,\tau\right)}{\partial s}\times\frac{\partial \mathbf{S}_1\left(s,\tau\right)}{\partial \tau}\right)\ , \tag{35}$$

where $\mathbf{S}_1\left(s,\tau\right)$ is a homotopy map between the side $(\mathbf{z},\mathbf{\Omega}_1)$ and the side $(\mathbf{z},\mathbf{S}_f)$ (29). This term therefore represents the area enclosed by the trajectory of the spin vector, and hence there is a corresponding two-form (22). Using Stoke's theorem, it can be written in terms of a path integral. It is also well known (22) that in this form the 1-form that results is physically the potential of a magnetic monopole at the center of a sphere. Since a full discussion of the topological nature of this term is outside the scope of this paper, we refer to the above literature for further details. The bath degrees of freedom are eliminated by a stationary-phase evaluation of the integral. The phase is an extremum for states that satisfy

$$i\partial_t\Phi_{1,k}\left(t\right)=\frac{\delta\mathcal{H}}{\delta\Phi^*_{1,k}(t)} \tag{36}$$

and a similar equation for $\Phi^*_{1,k}$. Summations over k are implicit in what follows. We have

$$i\partial_\tau\Phi_{1,k}(\tau)=\omega_k\Phi_{1,k}(\tau)-\gamma_k(\tau)S_- \tag{37}$$

$$i\partial_\tau\Phi^*_{1,k}(\tau)=\omega_k\Phi^*_{1,k}(\tau)-\gamma^*_k(\tau)S_+. \tag{38}$$

The solutions with the correct boundary conditions are

$$\begin{aligned}
\Phi_{i,k}(\tau) &= \Phi_{1,k}e^{-i\omega_k\tau}+ie^{-i\omega_k\tau}\int_0^\tau dt' e^{i\omega_k t'}\gamma_k(t')S_-(t') \\
\Phi^*_{i,k}(\tau) &= \Phi^*_k e^{i\omega_k(\tau-t)}+ie^{i\omega_k\tau}\int_\tau^t dt' e^{-i\omega_k t'}\gamma^*_k(t')S_+(t') \\
& \quad 0\le\tau\le t.
\end{aligned} \tag{39}$$

At the endpoints, we then have:

$$\begin{aligned}
\Phi_{1,k}(t) &= \Phi_{1,k}e^{-i\omega_k t}+ie^{-i\omega_k t}\int_0^t dt' e^{i\omega_k t'}\gamma_k(t')S_-(t') \\
\Phi^*_{1,k}(0) &= \Phi^*_k e^{-i\omega_k t}+i\int_0^t dt' e^{-i\omega_k t'}\gamma^*_k(t')S_+(t').
\end{aligned} \tag{40}$$

These solutions are then put back in Eq. (33). Similar expressions follow from the calculations of the backward propagator. The density matrix element at the initial time is calculated with the assumption that the bath is initially at equilibrium with the spin. The bath relaxes much faster than the spin, a reasonable approximation in many problems in magnetics. In this case the density matrix is separable at the initial time. The bath density matrix, ρ_B,

is then known and its matrix elements can be written explicitly in terms of the Hamiltonian, $\mathcal{H}_B$, of the bath only,

$$\begin{aligned}
\langle\ \Phi_1 \left|\rho_B(0)\right| \Phi_2\ \rangle \quad &= \quad \frac{1}{Z_B}\langle \Phi_1 \left|e^{-\beta \mathcal{H}_B}\right| \Phi_2\rangle \qquad (41)\\
&= \frac{1}{Z_B}\int_{\Phi_2}^{\Phi_1} \mathfrak{D}\Phi \exp\{\ \Phi^*(\beta)\Phi(\beta)\\
&+i\int_0^\beta d\tau\ [i\Phi^*(\tau)\Phi(\tau) - \mathcal{H}(\tau)]\ \}
\end{aligned}$$

with periodic boundary conditions

$$\begin{aligned}
\mathbf{\Phi}(0) \quad &= \quad \mathbf{\Phi}_2 \qquad (42)\\
\mathbf{\Phi}^*(\beta) \quad &= \quad \mathbf{\Phi}_2^*.
\end{aligned}$$

We find, after applying a stationary-phase approximation to the integral, the expression

$$\langle \Phi_1 \left|\rho_B(0)\right| \Phi_2\rangle = \exp\left\{\sum_k \Phi_{1,k}^* \Phi_{2,k} e^{-\beta\omega_k}\right\}. \qquad (43)$$

After integrating out the degrees of freedom of the bath, we are left with only integrals over paths in the spin configuration space. The effective action of the spin is now complex, as usual with dissipative systems. The reduced density matrix element is now given by

$$\begin{aligned}
\rho_{ff'}(t) \quad = \quad & \int \mathfrak{D}\mathbf{\Omega}_1 \int \mathfrak{D}\mathbf{\Omega}_2\ \langle \mathbf{\Omega}_1 \left|\rho_s(0)\right| \mathbf{\Omega}_2\rangle \qquad (44)\\
& \times \int_{\mathbf{\Omega}_1}^{\mathbf{S}_f} \mathfrak{D}\mathbf{S}_1 \int_{\mathbf{S}_{f'}}^{\mathbf{\Omega}_2} \mathfrak{D}\mathbf{S}_2 \exp\left\{iH_z \int_0^t dt'\left(S_{1,z}(t') - S_{2,z}(t')\right)\right.\\
& \left. + iS_{WZ}[\mathbf{S}_1] - iS_{WZ}[\mathbf{S}_2]\ \right\} \times \mathcal{W}(\mathbf{S}_1, \mathbf{S}_2)
\end{aligned}$$

where the last term is entirely due to the coupling between the bath and the spin particle. It is given by

$$\begin{aligned}
\ln \mathcal{W}(\mathbf{S}_1, \mathbf{S}_2) \quad &= -\int_0^t dt' \int_0^t dt''\ \exp\left(-i\omega_k (t'-t'')\right) \gamma_k^*(t')\, \gamma_k(t'') \qquad (45)\\
&\times \{\ \Theta(t''-t')\, S_{2,+}(t')\, S_{2,-}(t'')\\
&+ (1-\Theta(t''-t'))\, S_{1,+}(t')\, S_{1,-}(t'')\ \}\\
&+ \int_0^t dt' \int_0^t dt''\ \exp\left(-i\omega_k (t'-t'')\right) \gamma_k^*(t')\, \gamma_k(t'')\\
&\quad S_{2,+}(t')\, S_{1,-}(t'')\\
&+ \tfrac{1}{e^{\beta\omega_k}-1} \int_0^t dt' \int_0^t dt'' \exp\left(-i\omega_k (t'-t'')\right) \gamma_k^*(t')\, \gamma_k(t'')\\
&\times \{\ S_{1,+}(t')\, S_{2,-}(t'') + S_{2,+}(t')\, S_{1,-}(t'')\}\\
&- \tfrac{1}{e^{\beta\omega_k}-1} \int_0^t dt' \int_0^t dt'' \exp\left(i\omega_k (t'-t'')\right) \gamma_k(t')\, \gamma_k^*(t'')\\
&\times \{\ S_{1,+}(t'')\, S_{1,-}(t') + S_{2,+}(t'')\, S_{2,-}(t')\}\,.
\end{aligned}$$

By taking the limit of an infinite number of oscillators, this latter term becomes responsible for the appearance of dissipation in this model. After calculating the elements of the reduced density matrix, we can now calculate its time evolution and find a Fokker-Plank type equation as was done in the original work of CL (15). We choose rather to take the semiclassical limit of this expression and see under what conditions, if any, a LLGB equation can be recovered.

4 THE SEMI-CLASSICAL APPROXIMATION

In this section, we find the equation of motion of the magnetization by calculating the most probable configurational paths. This is done by calculating the path in the reduced density matrix element for the spin field that has the largest weight. Then we show that these paths are really the semiclassical limit of the classical paths averaged over the thermal fluctuations in the LLGB equation. We also show that the fluctuation-dissipation theorem is satisfied. It reduces to the Brown form only in the high temperature limit and only in the linear response approximation. This approximation fails when the system is highly anisotropic (10).

To facilitate the taking of the classical limit we make the change of variables,

$$\begin{aligned} \mathbf{S}(\tau) &= \tfrac{1}{2}\left(\mathbf{S}_1(\tau)+\mathbf{S}_2(\tau)\right), \\ \mathbf{D}(\tau) &= \mathbf{S}_1(\tau)-\mathbf{S}_2(\tau). \end{aligned} \tag{46}$$

The variable $\mathbf{D}$ represents the fluctuating field that is coupled to the spin and is due to the inherent irreversibility in the system. In terms of these new variables, the weight functional $\mathcal{W}(\mathbf{S}_1,\mathbf{S}_2)$ becomes

$$\begin{aligned} \mathcal{W}[\mathbf{S},\mathbf{D}] = \exp\Big\{ & \textstyle\int_0^t dt' \int_0^t dt'' e^{-i\omega_k t'} e^{i\omega_k t''} \gamma_k^*(t')\,\gamma_k(t'') \\ & \times\left\{-\tfrac{1}{2}D_+(t')\,D_-(t'') + \Theta(t''-t')\,S_+(t')\,D_-(t'')\right. \\ & \left. -\left(1-\Theta(t''-t')\right)D_+(t')\,S_-(t'')\right\} \\ & - \tfrac{1}{e^{\beta\omega_k}-1}\textstyle\int_0^t dt' \int_0^t dt'' e^{-i\omega_k t'} e^{i\omega_k t''}\left(D_+(t')\,D_-(t'')\right)\Big\} \end{aligned} \tag{47}$$

where $\Theta(\tau)$ is the unit step function

$$\Theta(\tau) = \frac{1}{2} + \frac{1}{2\pi i}\mathcal{P}\int_{-\infty}^{\infty} e^{i\omega\tau}\,\frac{d\omega}{\omega}. \tag{48}$$

In these new variables, the phase $ln\mathcal{W}$ clearly shows that the action of the reservoir results in an extra linear coupling in $\mathbf{S}$ and $\mathbf{D}$. Moreover, we now have

a quadratic term involving the variable $\mathbf{D}$. This quadratic term is easily seen to be real and negative, assuring convergence of the sum over all configurations of $\mathbf{D}$. The linear term describing interaction of the fields $\mathbf{S}$ and $\mathbf{D}$ is imaginary, however. In fact, it is such a term that gives rise to dissipation in the energy of the spin $\mathbf{S}$. As we will show below, $\mathbf{D}$ is the field that is associated with the classical random field in the LLGB equation. If the coupling term in $\ln \mathcal{W}$ is set to zero, then integrations over $\mathbf{D}$ are equivalent to averaging over a Gaussian fluctuating field. As will be seen below, this is the limit in which we recover the LLGB equation. In this case the bath does not depend on $\mathbf{S}$. This approximation fails when the Hamiltonian is not symmetric under the interchange of the dynamical spin components.

To find the semiclassical result for the reduced density matrix, we again resort to a stationary phase approximation to the phase of the path integrals in Eq.(44). First we impose constraints on the spin magnitude by introducing two Lagrangian multiplier fields, $\eta_1(t)$ and $\eta_2(t)$. These are then coupled to the spin fields $\mathbf{S}_1$ and $\mathbf{S}_2$, respectively, as follows,

$$\delta\left(\mathbf{S}_1^2(\tau)-1\right)=\int \mathfrak{D}\eta_1(\tau)\ e^{i\int d\tau\ \eta_1(\tau)\left(\mathbf{S}_1^2(\tau)-1\right)}. \tag{49}$$

A similar expression holds for the other spin vector variable, $\mathbf{S}_2$, and η_2. These constraints are then put back in the expression for the reduced density matrix element. The phase of the path integral is now a function of four independent fields η_1, η_2, $\mathbf{S}$ and $\mathbf{D}$. Extremizing the phase of these paths gives the semiclassical solution in the large spin limit. After solving for the constrained fields in terms of the spin fields $\mathbf{S}_1$ and $\mathbf{S}_2$, we write the remaining two equations in $\mathbf{S}$ and $\mathbf{D}$ only, obtaining a generalized form of the LLGB equation

$$\frac{d\mathbf{S}(t')}{dt'}=\mathbf{S}\times\left(\mathbf{H}+\mathbf{T}^{(S)}+\mathbf{T}^{(D)}\right)+\frac{1}{4}\mathbf{D}\times\mathbf{W}\ , \tag{50}$$

and

$$\frac{d\mathbf{D}(t')}{dt'}=\mathbf{D}\times\left(\mathbf{H}+\mathbf{T}^{(S)}\right)+\mathbf{S}\times\mathbf{W}+\mathbf{D}\times\mathbf{T}^{(D)}. \tag{51}$$

The vectors $\mathbf{T}^{(S)}$, $\mathbf{T}^{(D)}$ and $\mathbf{W}$ are associated with dissipation, thermal fluctuations and magnitude fluctuations, respectively. In Cartesian form, they are respectively given in terms of the functions J and F_β by

$$\mathbf{T}^{(S)}(u)=\begin{bmatrix} i\int_0^u dt'\Theta(u-t')\left\{J(u-t')S_-(t')-J^*(u-t')S_+(t')\right\} \\ -\int_0^u dt'\Theta(u-t')\left\{J(u-t')S_-(t')+J^*(u-t')S_+(t')\right\} \\ 0 \end{bmatrix}, \tag{52}$$

$$\mathbf{T}^{(D)}(u)=\begin{bmatrix} i\int_0^t dt'\left\{\left(\frac{1}{2}J(u-t')+F_\beta(u-t')\right)D_-(t')\right. \\ \left.+\left(\frac{1}{2}J^*(u-t')+F_\beta^*(u-t')\right)D_+(t')\right\} \\ \\ -\int_0^t dt'\left\{\left(\frac{1}{2}J(u-t')+F_\beta(u-t')\right)D_-(t')\right. \\ \left.-\left(\frac{1}{2}J^*(u-t')+F_\beta^*(u-t')\right)D_+(t')\right\} \\ \\ 0 \end{bmatrix}, \tag{53}$$

and

$$\mathbf{W}(u)=\begin{bmatrix} -i\int_0^t dt'\Theta(t'-u)\left\{J(u-t')D_-(t')-J^*(u-t')D_+(t')\right\} \\ \\ \int_0^t dt'\Theta(t'-u)\left\{J(u-t')D_-(t')+J^*(u-t')D_+(t')\right\} \\ \\ 0 \end{bmatrix}. \tag{54}$$

These vectors are in general non-local in time and hence include memory effects in the equations of motion for $\mathbf{S}$ and $\mathbf{D}$. This type of behavior is clearly needed when the relaxation time of the reservoir is of the same order as that of the spin particle. We will not discuss such a situation here. We are mainly interested to recover the constant dissipation case. Even though we called Eq.(50) and Eq.(51) generalized LLGB equations, it is *not* yet clear how a Gilbert damping term can arise in these equations. However through a careful choice of the density of states of the bath, the coupling constants and the initial conditions, such damping form can be recovered as shown below.

To describe dissipation, we take the continuum limit for the bath states. Then the spectral functions J and F_β are given by

$$J(\tau-\tau')=\int_0^\infty d\omega\ \lambda(\omega)\left|\gamma(\omega)\right|^2\exp\left[-i\omega(\tau-\tau')\right] \tag{55}$$

and

$$F_\beta(\tau-\tau')=\int_0^\infty d\omega\ \frac{\lambda(\omega)}{\exp\left[\beta\omega\right]-1}\left|\gamma(\omega)\right|^2\exp\left[-i\omega(\tau-\tau')\right]. \tag{56}$$

F_β is simply the nonzero temperature counterpart of J. $\lambda(\omega)$ is the density of states of the bath. In fact the function,

$$\mathcal{G}(\tau'-\tau)=\frac{1}{2}J(\tau'-\tau)+F_\beta(\tau'-\tau), \tag{57}$$

is the inverse of the free propagator of the field $\mathbf{D}$.

The vectors $\mathbf{S}$ and $\mathbf{D}$ are orthogonal as follows from the constraint equations, Eq.(49). Note that when $\mathbf{D}$ is set to zero, the density matrix becomes diagonal but the equation of motion for $\mathbf{S}$ will still have an extra term besides the precessional term that is due to the external field $\mathbf{H}$. This extra term $\mathbf{T}^{(S)}(u)$ clearly always has a damping effect. We conclude that it is the vector $\mathbf{S}$ that must be identified with the classical magnetization and that $\mathbf{D}$ is the part that gives rise to the thermal fluctuations in $\mathbf{S}$. Finally, we observe that the last term in Eq.(50) can not be recovered in the classical limit. This quantum mechanical term is not present in the LLGB equation and is beyond a classical linear-response treatment of the problem of fluctuations. It is of higher order in $\mathbf{D}$ and temperature-independent. It is easy to see that this term gives rise to fluctuations in the magnitude of the spin. These fluctuations can not be accounted for classically since the magnitude of the magnetization is assumed to be constant.

One important thing to note from Eq.(50) is that the vector $\mathbf{T}^{(D)}$ is complex and hence, if fluctuations are present, the equation of motion for $\mathbf{S}$ becomes complex. The physical interpretation of this equation then becomes obscure at this level and may not be used as it stands to get the effective magnetization of the particle. Having a complex equation for the extremum path of the spin is however expected given that the same result happens in the case of the harmonic oscillator (15). A solution for the fluctuating magnetization is then sought through a direct calculation of the propagators in Eq.(33).

Now we show that a generalized fluctuation-dissipation theorem is satisfied as expected for this system since we started from a closed system and integrated out a large part of its degrees of freedom. We will also show that it is the vector $\mathbf{D}$ that should be regarded as the quantum source of the thermal fluctuations in the spin system as treated in LLGB. The vector $\mathbf{S}$ is then the physically measurable magnetization. To better understand the physical meaning of the field $\mathbf{D}$ and to recover the standard stochastic description of the thermal fluctuations, we introduce yet another field, $\xi(t)$. In Eq.(33), we will replace the l.h.s. of the following expression with the r.h.s.

$$exp\left(-\int_0^t dt' \int_0^t dt'' \mathbf{D}_+(t')\, \mathcal{G}(t'-t'')\, \mathbf{D}_-(t'')\right) = \tag{58}$$

$$\mathcal{N}\int \mathfrak{D}\xi\; exp\left(-\frac{1}{2}\int_0^t dt' \int_0^t dt'' \xi_l(t')\, \frac{1}{2}\mathcal{G}_{ll}^{-1}(t'-t'')\, \xi_l(t'') - i\int_0^t dt' \xi_l(t')\, \mathbf{D}_l(t')\right)$$

where $\mathcal{N}$ is a normalization constant. The quadratic term in $\mathbf{D}$ is then assumed to be a result of an averaging over all configurations of the field ξ. Hence the path-integral for the reduced density, Eq.(33), is now in terms of three fields. The field ξ will now result in a third equation of motion. However to recover a thermal field similar to that introduced by Brown (1), we proceed by assuming

that the field ξ is classical, i.e., we ignore its equation of motion. At this point the fields ξ and $\mathbf{H}$ are treated as non-dynamical fields. Next we solve for the magnetization $\mathbf{S}$ for a given ξ and only then do we average over all configurations of ξ with the quadratic weight that we originally ignored in the solution. In fact ξ becomes the Brown stochastic field if we take the classical limit, that of high temperature. Given these observations, we can now assume that the effective thermal field with which the spin is interacting is really nothing more than the abstract $\mathbf{D}$ field that has been introduced in this calculation of the reduced density matrix element. In fact ξ has the following correlation functions

$$\begin{aligned} \langle \xi_l(\tau)\, \xi_{l'}(\tau') \rangle &= 2\delta_{ll'} \mathcal{G}(\tau - \tau') \\ &= \frac{1}{\pi} \delta_{ll'} \int_0^\infty d\omega\, \omega \coth\left(\frac{\beta\omega}{2}\right) \frac{\pi \lambda(\omega) \left|\gamma(\omega)\right|^2}{\omega} \exp\left[-i\omega(\tau - \tau')\right]. \end{aligned} \tag{59}$$

To recover the correlations of the thermal field assumed in the LLGB equation, we simply take the high temperature limit and require that the bath satisfies the condition,

$$\frac{\pi \lambda(\omega) \left|\gamma(\omega)\right|^2}{\omega} = \alpha, \tag{60}$$

where α is a constant. This condition provides the simplest relation between fluctuations and dissipation. In this case, the correlation functions for the random field become simply

$$\langle \xi_l(\tau)\, \xi_{l'}(\tau') \rangle = 2\delta_{ll'} \alpha k T \delta(\tau' - \tau). \tag{61}$$

A similar condition arises if we replace the spin degrees of freedom by those of a harmonic oscillator (15). However at high temperature, as we noticed earlier, a large spin can be approximated well by an oscillator. This condition is, however, still true even if the bosonic degrees of freedom of the bath are replaced by fermionic degrees of freedom.

Finally we consider recovering the LLG equation with the Gilbert form of damping. Equations (50) and (51) are very general as they stand and it is not clear if the dissipation has the Gilbert form. To deduce the very special case of constant damping with the Gilbert form, we set the fluctuations to zero and take the following form for the spectral function J,

$$J(\tau' - \tau) = i\alpha \frac{d}{dt} \delta(\tau' - \tau). \tag{62}$$

After an integration by parts of the term containing $\mathbf{T}^{(S)}$ in the reduced density matrix element, Eq.(33), the equation of motion for $\mathbf{S}$, the magnetization, becomes simply

$$\frac{d\mathbf{S}(\tau)}{d\tau} = \mathbf{S}(\tau) \times \left(\mathbf{H} + \alpha\left(\frac{d\mathbf{S}(\tau)}{d\tau} - \frac{d\, \mathbf{S}\cdot\mathbf{z}}{d\tau}\mathbf{z}\right)\right) \tag{63}$$

The boundary terms in the integration by parts are easily dealt with by a renormalization of the measure of the path-integral. Keeping in mind the model used to derive this result, this equation reduces to the LLG form only in the limit of small deviations from local equilibrium. It is also important to note that the choice we made for J, Eq.(62), is compatible with the Gaussian approximation for the thermal field. Hence the stochastic LLG equation is compatible with the FDT at high temperatures. However in this case, the application of the FDT to the LLG equation is not so trivial as for the generalized LLGB equations, Eqs. (50) , (51). In this case the correlation functions will depend on the dynamics of the system, i.e., the symmetry of the Hamiltonian.

Before we end this section, we make a final comment about the condition, Eq.(60), by which we recovered the LLG limit. If we assume constant coupling constants for the interaction between the spin and the bath, we find that the density of states must be linear. For phonons, the density of states is quadratic and hence, based on this assumption, can not be the major source of the damping constant α. In fact dissipation due to currents is believed to be much larger (30). Ferromagnetic compounds, such as FeNi, show a complex density of states for the non-localised electrons, hence a condition such as that given in Eq.(60) is representative of many competing mechanisms. It is only the lower part of the spectrum that is important for a constant dissipation. In fact in Eq.(55), the limit of integration can not be taken to be infinite for a real bath. This in turn will introduce a new cut-off parameter in condition Eq.(60) which will be system dependent.

5 ANISOTROPIC PARTICLES

Perpendicular recording requires particles with relatively high anisotropy for long-term storage purposes. Hence a relevant question to ask is how does anisotropy interact with the thermal field. This requires a treatment beyond the linear response approach. We treat this question in this last section. We limit ourselves to the simplest case; that of uniaxial anisotropy along the z-axis. In this case the Hamiltonian of the particle-bath system becomes

$$\mathcal{H} = -H_z S_z - K S_z^2 + \sum \omega_k a_k^+ a_k - \sum \gamma_k a_k^+ S_- - \gamma_k^* S_- a_k, \qquad (64)$$

where K is the anisotropy constant. The external field is taken along the easy axis. Similarly to the above calculation, we find that the reduced density matrix elements in the presence of anisotropy become

$$
\begin{aligned}
\rho_{ff'}(t) &= \int \mathfrak{D}\boldsymbol{\Omega}_1 \int \mathfrak{D}\boldsymbol{\Omega}_2 \, \langle \, \boldsymbol{\Omega}_1 \left| \rho_s(0) \right| \boldsymbol{\Omega}_2 \, \rangle \qquad (65)\\
&\times \int_{\boldsymbol{\Omega}_1}^{\mathbf{S}_f} \mathfrak{D}\mathbf{S}_1 \int_{\mathbf{S}_{f'}}^{\boldsymbol{\Omega}_2} \mathfrak{D}\mathbf{S}_2 \exp\left\{ iH_z \int_0^t dt' \, (\, S_{1,z}(t') - S_{2,z}(t')) \right.\\
&\left. + iK \int_0^t dt' \left(S_{1,z}^2(t') - S_{2,z}^2(t') \right) + i \, (\, S_{WZ}[\mathbf{S}_1] - S_{WZ}[\mathbf{S}_2]) \right\} \mathcal{W}(\mathbf{S}_1, \mathbf{S}_2) \, .
\end{aligned}
$$

Clearly the bath influence on the magnetic moment with and without anisotropy is the same as before, but now there is coupling between the fluctuating field and the spin field that is anisotropy-dependent. Therefore the random field becomes K -dependent beyond the linear-response approximation. We show below how to calculate the new correlation functions of the fluctuations beyond the Gaussian approximation. We define two new anistropy related vectors,

$$
\mathbf{K}_S = \widehat{z}\,(2KS_z) \ , \qquad (66)
$$

and

$$
\mathbf{K}_D = \widehat{z}\,(2KD_z) \, . \qquad (67)
$$

The equations of motion for the spin field and the fluctuating field become in this case

$$
\frac{d\mathbf{S}(t')}{dt'} = \mathbf{S}\times \left(\mathbf{H} + \mathbf{K}_S + \mathbf{T}^{(S)} + \mathbf{T}^{(D)} \right) + \frac{1}{4}\mathbf{D} \times (\mathbf{W} + \mathbf{K}_D) \, , \qquad (68)
$$

and

$$
\frac{d\mathbf{D}(t')}{dt'} = \mathbf{D} \times \left(\mathbf{H} + \mathbf{K}_S + \mathbf{T}^{(S)} \right) + \mathbf{S} \times (\mathbf{K}_D + \mathbf{W}) + \mathbf{D} \times \mathbf{T}^{(D)} \, . \qquad (69)
$$

We observe that the additive terms on the right now become K-dependent. Hence the fluctuations of the magnitude of the magnetization are anisotropy-dependent. The equation of motion for $\mathbf{D}$ shows that anisotropy dependence can be in the precessional term and hence can be recovered even in the classical limit beyond a linear-response approach. We next show that this indeed the case.

From Eq.(65), the phase of the path integral, which we denote by $\boldsymbol{\Gamma}$, is identified with the effective action for the spin system. The procedure to find correlation functions is standard (31). To this action we add two independent external sources $\mathbf{Q}_1$, $\mathbf{Q}_2$. These sources will then be coupled to the fields $\mathbf{S}_1$ and

$\mathbf{S}_2$, respectively, and will be used to generate two-point correlation functions for the fields $\mathbf{S}$ and $\mathbf{D}$. Since in the following we restrict ourselves to equilibrium properties, the sources $\mathbf{Q}_1$ and $\mathbf{Q}_2$ will be taken as equal, but still arbitrary. This is equivalent to making the external field $\mathbf{H}$ arbitrary. Hence $\mathbf{H}$ will be used instead to generate the correlation functions. Since we are solely interested in how the correlation functions depend on the anisotropy constant K, at the end we take the limit $\mathbf{H} \to 0$. We also restrict the discussion to small deviations of the magnetization from the $z-$axis.

First, we define a new functional $\mathbb{Z}$ of the external field $\mathbf{H}$,

$$\begin{aligned} \mathbb{Z}\left[\mathbf{H}\right] &= \int \mathfrak{D}\mathbf{S}_f \mathfrak{D}\mathbf{S}_{f'} \int \mathfrak{D}\mathbf{\Omega}_1 \mathfrak{D}\mathbf{\Omega}_2 \langle \mathbf{\Omega}_1 | \rho_s(0) | \Omega_2 \rangle \\ &\quad \times \int \mathfrak{D}\mathbf{S}_1 \mathfrak{D}\mathbf{S}_2 \ \exp\left[\, \mathbf{\Gamma}\left[\, \mathbf{H} \,\right]\,\right]. \end{aligned} \tag{70}$$

This functional is defined in such a way that its variations with respect to $\mathbf{H}$ generate all correlation functions of the fields $\mathbf{S}$ and $\mathbf{D}$ at equilibrium. The average value of the field $\mathbf{D}$ is clearly given by

$$\left. \frac{1}{\mathbb{Z}} \frac{\delta \mathbb{Z}}{\delta H_\alpha\left(t\right)} \right|_{\mathbf{H} \longrightarrow 0} = i \left\langle D_\alpha\left(t\right) \right\rangle . \tag{71}$$

The two-point irreducible correlation function is similarly given by

$$\begin{aligned} \left. \frac{1}{\left(i\right)^2} \frac{\delta^2 ln \mathbb{Z}}{\delta H_\alpha\left(t\right) \delta H_\beta\left(t'\right)} \right|_{\mathbf{H} \longrightarrow 0} &= \left\langle D_\alpha\left(t\right) D_\beta\left(t'\right) \right\rangle - \left\langle D_\alpha\left(t\right) \right\rangle \left\langle D_\beta\left(t'\right) \right\rangle \\ &\equiv \mathcal{G}^{-1}_{\alpha\beta}\left(t - t'\right). \end{aligned} \tag{72}$$

From the equations of motion of the fields $\mathbf{S}$ and $\mathbf{D}$, we deduce the full equation of motion of the function $\mathcal{G}^{-1}_{\alpha\beta}$,

$$\begin{aligned} \frac{\partial}{\partial t} \mathcal{G}^{-1}_{\alpha\gamma}\left(t - t'\right) &= \epsilon^{\alpha\beta\lambda} \mathcal{G}_{\beta\gamma}\left(t - t'\right) H_\lambda + \epsilon^{\alpha\beta\lambda}\left(2K\right) \mathcal{G}_{\beta\gamma}\left(t - t'\right) \delta_{\lambda 3} \langle S_\lambda\left(t\right) \rangle \\ &\quad + \epsilon^{\alpha\beta\lambda}\left(2K\right) \delta_{\lambda 3} \mathcal{G}_{\lambda\gamma}\left(t - t'\right) \left\langle S_\beta\left(t'\right) \right\rangle + i \epsilon^{\alpha\beta\lambda} \mathcal{G}_{\beta\gamma}\left(t - t'\right) \left\langle T^s_\lambda\left(t\right) \right\rangle \\ &\quad + i \epsilon^{\alpha\beta\lambda} \langle S_\beta\left(t'\right) \rangle \frac{\delta}{i \delta H_\gamma\left(t\right)} \left\langle W_\lambda\left(t\right) \right\rangle , \end{aligned} \tag{73}$$

where $\epsilon^{\alpha\beta\gamma}$ is the antisymmetric unit tensor. In the following the quantum terms are neglected and we look for corrections to $\mathcal{G}^{-1}$ due only to the anisotropy term. We solve Eq.(73) by iteration, starting from the free $\mathcal{G}^{-1}$ propagator. We find that the off-diagonal terms are now non-zero and depend explicitly on K. For the 1-2 element of $\mathcal{G}^{-1}$, we find that

$$\frac{\partial}{\partial t} \mathcal{G}^{-1}_{12}\left(t - t'\right) = A \frac{K}{T} \delta\left(t - t'\right) \tag{74}$$

where A is a constant proportional to the magnetization. To obtain this equation we also assumed that the average of the vector $\mathbf{D}$ is zero. This assumption is due to the fact that the average of the classical stochastic field, ξ, is also taken to be zero.

We define $\widetilde{\mathcal{G}}^{-1}$, the Fourier Transform of $\mathcal{G}^{-1}$, using

$$\mathcal{G}_{12}^{-1}\left(t-t'\right) = \frac{1}{2\pi}\int d\omega\ e^{i\omega\left(t-t'\right)}\widetilde{\mathcal{G}}_{12}^{-1}\left(\omega\right)\ . \tag{75}$$

Solving in frequency Fourier space, we find that

$$\widetilde{\mathcal{G}}_{12}^{-1}\left(\omega\right) = \frac{-iAK/T}{\alpha\omega}\ . \tag{76}$$

The other terms behave similarly as a function of K/T. Since within the above assumptions, the correlation functions of the classical thermal field are the inverse of those of $\mathbf{D}$-field, we conclude that thermal fluctuations and anisotropy behave oppositely. The less the anisotropy the higher the level of fluctuations. Higher order corrections can similarly be calculated in this manner.

6 CONCLUSION

Using coherent states and a simple quantum mechanical model for a single large spin particle, we have shown that a generalized form of the Landau-Lifshitz equation can be recovered in the limit of high temperature. In this case the damping constant provides relaxation to the local equilibrium state. We have also shown how fluctuations give two different contributions to the magnetization. One contribution is magnitude conserving and the other is not. We derived generalized equations for the magnetization that include non-local effects in the dissipation term and go beyond the simple linear-response approach. An important immediate result of this work is the dependence of fluctuations on the anisotropy of the system. The LLGB equation is clearly inadequate in this respect. However these deficiencies can be corrected by using the right correlation functions for the fluctuations. Changing the damping to a tensor quantity in the LLG equation (10) to account for noise correctly is then not needed. More complicated couplings, other than the linear coupling considered here, naturally induce a tensor character for the relaxation. Most of these conclusions are true even in the classical limit. Garcia-Palacios (32) treated similar questions at the classical level and our results agree in that limit. Generalizing our model to the anisotropic case does not change these conclusions. In the linear approximation, the results of Safonov and Bertram (10) are reproduced from our results by taking the vector $\mathbf{D}$ to be the stochastic field. However the results of Smith and Arnett (7) are found by taking the time derivative of the vector $\mathbf{D}$ to be the random force. Clearly, as seen from Eq.(51), in this latter case the correlation functions of the random force can not be taken to be

independent of the dynamics of the system. An isotropic fluctuation is simply not accurate enough for an asymmetric Hamiltonian. More details on this comparison will be communicated elsewhere.

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